Risky Business

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We all realize that our evaluations can be no better than the data, and model, allow. At the simplest level we often select Optimistic, Expected and Pessimistic parameter estimates, and bound the result accordingly.

It is, however, relatively simple to address the uncertainty question in a more comprehensive, quantitative fashion, and better identify ***where to focus time, and money, in search of an improved evaluation.***

As carbonate (rather than shaly sand) petrophysicists, ***our Sw estimates are typically compromised by uncertainty in the Archie equation attributes***.

$$S_w^{\,n} = a\, R_w / (\Phi^{m} R_t)$$

By taking the derivative of Archie's equation (the same approach will suffice for a shaly sand equation), one is able to quantify the individual impact of each term upon the result, and thus ***recognize where the biggest bang for the buck, in terms of a core analyses program or suite of potential logs, is to be found*** (Figure 1).

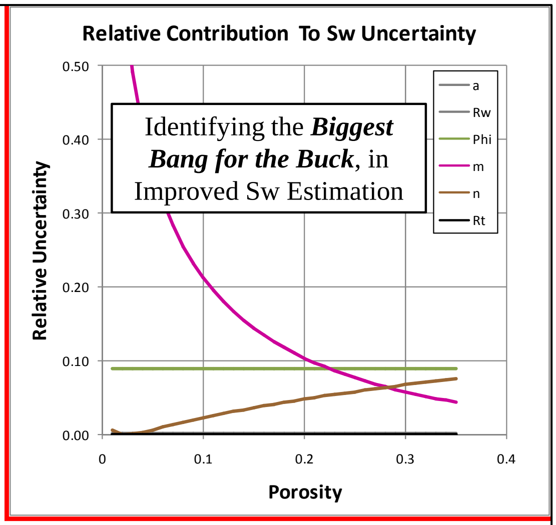
Figure 1

•***Below***: Illustrative Best Estimate of each parameter, with corresponding individual uncertainty, and associated relative uncertainty on Sw(Archie), *at a specific porosity.*

•***Right***: ***Relative impact on Sw(Archie) uncertainty of 'm' & 'n', across a range of porosity values, for a fixed Phi uncertainty.***

Attribute Uncertainties Specified Individually			
Light Green Cells require User Specification			
Light Blue Cells are calculated results			
Attribute	Individual Uncertainty	Best Estimate	Relative Uncertainty On Sw(Archie)
a	0.0%	1.00	0.0000
Rw	4.4%	0.02	0.0019
Phi	15.0%	0.20	0.0900
m	10.0%	2.00	0.1036
n	5.0%	2.00	0.0480
Rt	1.0%	40.00	0.0001
Sw		11%	
Sw^n		1%	
Sw^n=0.367 is an inflection point			

After C. Chen and J. H. Fang. Sensitivity Analysis of the Parameters in Archie's Water Saturation Equation. The Log Analyst. Sept – Oct 1986



•The relative importance of 'm' and 'n' depend not only upon their specific uncertainty, but also upon the porosity of the interval in question; there is a link

•Uncertainty resulting from 'a', R_w and R_t is below that of ϕ, 'm' & 'n' in this illustrative example

Issues

We've all worked Fields for which we had high confidence in $\boldsymbol{R_w}$, but that is not always the case. R_w is the first example of an important input parameter, which is usually subject to (at least some) uncertainty.

In some (certain areas of the Shuaiba, for example: Ballay, 2001) reservoirs, ***mineralogy*** is nearly uniform, and hence well known. In other (probably most) locales, uncertainty exists in this basic information, which will carry through to porosity. Furthermore, one may be faced with three minerals (limestone – dolostone – anhydrite) and only two logging tools (density-neutron), with anhydrite present as obvious nodules, or as (more subtle) cement

Porosity is often thought of as +/- "x" pu of uncertainty, but in actual fact the uncertainty is mineral composition dependent, and may also be a function of the amount of porosity present (some measurements are more accurate at high porosity, and vice versa). Uncertainty is compounded by tool type, era (old versus modern) and borehole conditions. Additionally (Adams, 2005 & Denney, 2005), uncertainty ranges should recognize the possibility of an inappropriate model.

Mud resistivity (and hence borehole effects) may change from one well to the next, indeed one logging interval to the next. The 6FF40, as an example, has a Skin Effect limitation at the low resistivity end (~ 1 ohm-m), and a signal-to-noise limitation at high resistivity (~100 ohm-m). Mud filtrate invasion, parallel (induction) versus series (laterolog) circuit issues, vertical tool resolution, etc compound the uncertainty in ***resistivity***. There are a number of inter-related issues, with additional details to be found in George (2003).

How many of us have ever been "really sure" of our ***'m' and 'n'*** exponents? Focke and Munn (1987) nicely illustrate the dependence of 'm' upon carbonate pore geometry, while 'n' is controlled by wettability (Sweeny & Jennings, 1960) and surface roughness (Diederix, 1982). In carbonates, wettability (and hence "n") may vary with pore size (Chardac et al, 1997), and present an additional challenge, particularly in the transition zone.

Sw(Archie) involves the combination of all these attributes, and their respective uncertainties. Is it any wonder that "***one size may not fit all feet***"?

Einstein advised us to 'keep it as simple as possible, but no simpler'. With this in mind we ***estimate the uncertainty upon the Sw estimate, resulting from the individual attributes, in a deterministic form, which can be easily coded into spreadsheet, or petrophysical s/w***.

The Derivative

I always appreciate my geology friends providing me with supplemental details, and so given the wide backgrounds going into this question, it's worthwhile defining, and illustrating, ***what the (calculus) derivative, which is fundamental to this issue, represents.***

Differentiation is the mathematical technique for characterizing the rate at which a dependent variable (say "y") changes, as some independent variable (say "x") changes: the ***rate of that change is known as the derivative***. As a specific example, ***velocity is the*** (derivative) ***rate of change of position***, and ***acceleration is the*** (derivative) ***rate of change of velocity***.

At the simplest, linear level

$$y = m * x + b$$

with ‘m’ the slope and ‘b’ the intercept. When ‘x’ changes by some Δx, ‘y’ changes by Δy, according to the following.

$$y + \mathbf{\Delta y} = m * (x + \Delta x) + b = y + \mathbf{m\ \Delta x}$$

where we have inserted the relation y = m * x + b. The derivative of ‘y’, or the rate of change in ‘y’ as ‘x’ changes, is then ‘m’.

In the case of Sw(Archie) estimates, there are multiple independent attributes, and one then resorts to ***‘partial’ derivatives, which sum to characterize the total variation of the dependent estimate, as a function of the individual partial derivatives.***

Additional details, and illustrations, are to be found at the below link.

http://en.wikipedia.org/wiki/Derivative_(calculus)

http://en.wikipedia.org/wiki/Issac_Newton

Propagation of Error

Each of the Sw input attributes has a Best Estimate value, and an associated uncertainty distribution. The ***individual uncertainties ‘propagate’ through to the composite result according to a specific protocol***: http://en.wikipedia.org/wiki/Propagation_of_uncertainty. The uncertainty of each parameter may be characterized by the respective standard deviation (σ_x), which is the positive square root of the variance [$(\sigma_x)^2$]. As an example, the 68% confidence limits of a normally (bell-shaped) distributed variable ‘x’, are x +/- σ_x.

In general, the uncertainty in ‘y’, which is some function of variables x_1 ➔ x_n, is

$$(\sigma_y)^2 = \Sigma\ [\ (dy\ /\ dx_i)\ \sigma x_i\ ]^2$$

where the partial derivative of y, with respect to x_i, is represented by dy / dx_i, rather than the mathematically correct symbol, so as to minimize the need for special word processor symbols. See the Wikipedia reference, above, for more background information and the correct symbolism; there are additional mathematical details in Appendix V of the Mechanics Lab Manual, Case Western Reserve University (available on-line): Uncertainty and Error Propagation.

As a ***specific example of error propagation***, Ohm’s Law relates Resistance (R), Voltage (V) and Current (I) as

$$R = V / I$$

Measurement of both “V” and “I” are subject to uncertainty, which propagates through to “R”, according to (as above)

$$\Delta R^2 = (\Delta V / I)^2 + (V\ \Delta I / I^2)^2$$

Relative Uncertainty in Sw(Archie) Estimates

With an understanding of what the derivative means, and how individual uncertainties propagate, we proceed to ***determine the contribution of specific parameter uncertainties, to an S_w estimate.*** Those not interested in details may skip the following and go direct to Figure 2.

$$S_w = [\, R_w / (\phi^{\,m}\, R_t)\,]^{\,1/n} = R_w^{\,1/n}\, \phi^{\,-m/n}\, R_t^{\,-1/n}$$

Differentiating S_w with respect to R_w, ϕ, and R_t is straight-forward differentials, and yields

$$\mathbf{R_w \text{ partial derivative : } (1/n)\,[\, R_w^{(1/n-1)}\, \phi^{-m/n}\, R_t^{-1/n}\,] \Rightarrow S_w / (n\, R_w)}$$

$$\mathbf{\phi \text{ partial derivative : } (-m/n)\,[\, R_w^{1/n}\, \phi^{(-m/n-1)}\, R_t^{-1/n}\,] \Rightarrow -m\, S_w / (n\, \phi)}$$

$$\mathbf{R_t \text{ partial derivative : } (-1/n)\,[\, R_w^{1/n}\, \phi^{-m/n}\, R_t^{(-1/n-1)}\,] \Rightarrow -S_w / (n\, R_t)}$$

If an ‘a’ parameter is (locally) used in the Archie relation {ie $S_w = [\, a\, R_w / (\phi^{\,m}\, R_t)\,]^{\,1/n}$}, we recognize that its behavior is functionally similar to R_w, and the ‘a’ dependence is determined by analogy to the R_w expression.

The ***exponential dependence of S_w(Archie) upon “m” and “n” requires additional differentiation protocols***, as follows.

Differentiation of Natural Logarithms; consider a function f(x)

$$f(x) = x^2 + x + 1$$

The derivative of the natural logarithm of that function is

$$d\,[\ln f(x)\,]/dx = f\,'(x) / f(x)$$

As an illustration

$$d\,\{\ln[x^2 + x + 1]\,\}/dx = \{d\,[x^2 + x + 1]/dx\,\} / \{x^2 + x + 1\}$$

$$d\,\{\ln[x^2 + x + 1]\,\}/dx = \{2x + 1\} / \{x^2 + x + 1\}$$

The Chain Rule for Differentiation; consider a function f(u) , wherein “u” is some function of “x”

The derivative of f(u) with respect to “x” is (per the Chain Rule)

$$d\,f(u) / dx = [\, d\,f(u) / du\,]\,[\, du / dx\,]$$

As an illustration

$$y = f(u) = 4\,(2x - 7) = 4u \text{ with } u = 2x - 7$$

$$dy / dx = [\, dy/du\,]\,[\, du/dx\,] = 4 * 2 = 8$$

Implicit Differentiation; some mathematical relations involve an implicit relation, wherein the variable (or dependence) of interest is not expressed as a function of all other attributes. The dependence is "implicit"

Consider, for example, the equation of a circle, and for illustration purposes perform both explicit and implicit differentiation

$$x^2 + y^2 = 25$$

Explicitly (for reference)

$$y = +/- [25 - x^2]^{1/2}$$

$$dy/dx = (+/-)\ x / [25 - x^2]^{1/2}$$

As an example of implicit differentiation, take the derivative without explicitly solving for "y"

$$x^2 + y^2 = 25$$

Let f(x) = y, so that $f^2(x) = y^2 = 25 - x^2$

Implicitly (for illustration)

$$d[f^2(x)]/dx = 2 f(x) [df(x)/dx]$$

Solve the above relation for df(x)/dx to find the following

$$dy/dx = \{ d[f^2(x)]/dx \} / [2 f(x)] =$$

$$(+/-) [2x] / \{ 2 [25 - x^2]^{1/2} \} = x / [25 - x^2]^{1/2}$$

which is the same result as found with explicit differentiation in the preceding exhibit (thereby validating the implicit relation, which we will need below).

Drawing upon the preceding differentiation rules, determine the effect of 'n' on Sw uncertainty as follows (remember, to simplify the word processor notation, d/dx symbolizes partial differentiation)

$$d[\ln f(x)/dx] = [df(x)/dx] / f(x) \Rightarrow df(x)/dx = f(x)\ d[\ln f(x)/dx]$$

$$S_w = [R_w / (\phi^m R_t)]^{1/n}$$

$$\ln(S_w) = (1/n) \ln [R_w / (\phi^m R_t)] = u \ln [R_w / (\phi^m R_t)] \text{ where } u = 1/n$$

$$dS_w/dn = (dS_w/du)\ (du/dn) = (S_w) \ln[R_w / (\phi^m R_t)]\ (-1/n^2)$$

Recognizing that $\ln[R_w / (\phi^m R_t)]\ (-1/n^2) = (-1/n)\ln[Sw]$, one is left with

$$\mathbf{dS_w/dn = (S_w)[(-1/n)\ln(Sw)]}$$

Differentiating with respect to 'n' yields ***the effect of 'm' on S_w uncertainty***

$$d[\ln f(x)/dx] = [df(x)/dx] / f(x) \Rightarrow df(x)/dx = f(x)\ d[\ln f(x)/dx]$$

$$S_w = [R_w / (\phi^m R_t)]^{1/n}$$

$$\ln(S_w) = \ln [R_w / R_t]^{1/n} + \ln [\phi^{-m}]^{1/n} = \ln [R_w / R_t]^{1/n} - (m/n) \ln [\phi]$$

$$\ln(S_w) = \ln [R_w / R_t]^{1/n} - u \ln [\phi] \text{ where } u = m/n$$

$$\mathbf{dS_w/dm} = (dS_w/du)\ (du/dm) = \mathbf{(S_w) \ln[\phi]\ (-1/n)}$$

In addition to Chen (1986) and Bowers (2000), Carlos Torres-Verdin points out that Philippe Theys has investigated this issue as a topic in a book, and George Eden of BP Canada brought to my attention Burnie's 2004 article, which is also 'worth the read'. For our shaly sand colleagues, uncertainty in Sw(Shaly Sand) has been examined by Freedman (1985).

Recalling that the uncertainty in 'y', which is some function of variables $x_1 \rightarrow x_n$, is a result of propagating errors, according to

$$(\sigma_y)^2 = \Sigma [(dy / dx_i) \sigma x_i]^2$$

we are now able to ***quantify the uncertainty in S_w and the role that 'a', R_w, ϕ, 'm', 'n' and R_t are playing***, by squaring and summing the above partial derivatives, multiplied by their individual standard deviations. The ***relative uncertainty, one parameter to the next is simply the individual partial derivative multiplied by the respective standard deviation, squared***.

Following Chen (1986) the ***respective uncertainties are thought of as percentage wise specifications*** ("y%" in Chen's notation, ***in general unique for each attribute***, but ***uniformly symbolized as y% in the notation***).

$$dS_w / dR_w = S_w / (n * R_w)$$

$$(dS_w / dR_w)^2 (\sigma R_w)^2 = (S_w / n)^2 [(\sigma R_w)/ R_w]^2 = (S_w / n)^2 [(y\% R_w) / R_w]^2 = (S_w / n)^2 (y\%)^2$$

$$dS_w / da = S_w / (n * a)$$

$$(dS_w / da)^2 (\sigma a)^2 = (S_w / n)^2 [(\sigma a)/ a]^2 = (S_w / n)^2 [(y\% a) / a]^2 = (S_w / n)^2 (y\%)^2$$

$$dS_w / dR_t = S_w / (n * R_t)$$

$$(dS_w / dR_t)^2 (\sigma R_t)^2 = (S_w / n)^2 [(\sigma R_t)/ R_t]^2 = (S_w / n)^2 [(y\% R_t) / R_t]^2 = (S_w / n)^2 (y\%)^2$$

$$dS_w / d\phi = - m S_w / (n \phi)$$

$$(dS_w / d\phi)^2 (\sigma\phi)^2 = (S_w / n)^2 [m (\sigma\phi)/ \phi]^2 = (S_w / n)^2 [m (y\% \phi) / \phi]^2 = (S_w / n)^2 (m\ y\%)^2$$

$$dS_w/dm = (S_w) \ln[\phi] (-1/n)$$

$$(dS_w / dm)^2 (\sigma m)^2 = (S_w / n)^2 [\ln(\phi) (\sigma m)]^2 = (S_w / n)^2 [\ln(\phi)(y\% m)]^2 = (S_w / n)^2 [m \ln(\phi)\ y\%)]^2$$

$$dS_w/dn = (S_w) \ln(Sw) (-1/n)$$

$$(dS_w / dn)^2 (\sigma n)^2 = (S_w / n)^2 [\ln(S_w) (\sigma n)]^2 = (S_w / n)^2 [\ln(S_w)(y\% n)]^2 = (S_w / n)^2 [n \ln(S_w)\ y\%)]^2$$

Remember, in the symbolism, uncertainty for each attribute is written simply as y%, but the specific value is in general unique to each parameter.

Each of the terms share the $(S_w / n)^2$ prefix; dropping this results in the ***relative magnitudes of the various components***, as follows (again, using Chen's notation, and keeping in mind that ***in general, the individual y% have different specific values***).

$$C(R_w) = (y\%)^2$$

$$C(a) = (y\%)^2$$

$$C(R_t) = (y\%)^2$$

$$C(\phi) = (m\ y\%)^2$$

$$C(m) = [m \ln(\phi)\ y\%)]^2$$

$$C(n) = [n \ln(S_w)\ y\%)]^2$$

For complete clarity and comparison, Chen's 1986 definitions appear in Figure 2.

These equations reveal that 'a', R_w and R_t are dependent upon only their respective uncertainties, whereas ϕ, 'm' and 'n' involve other attributes. ***When the respective uncertainties are equal, one then has $C(\phi) > C(R_w), C(a), C(R_t)$*** simply because 'm' > 1: Figure 3. In such a situation, ***time and money are better spent on improved porosity estimates, rather than 'a', R_w or R_t.***

Because the ***relative uncertainty in 'm' and 'n' involves the*** (square of a) ***natural logarithm***, the behavior is more complicated, and involves ***an inflection point***: Figure 4.

Figure 2

•Denote ***percentage-wise uncertainty as "y%".*** This is, ***in general, unique to each parameter***

•'a', R_w and R_t all enter the Archie equation free of the 'm' exponent and share a similar result

•Porosity is raised to the 'm' power, and so the functional form is different from 'a', R_w and R_t

•The 'm' and 'n' exponents share a similar result, which is different than the other attributes

•In contrast with 'a', R_w and R_t, ***the relative importance of porosity, 'm' and 'n' <u>depend not only upon their specific uncertainty, but also upon other attributes.</u>***

$$C_a = \left(\frac{\sigma_a}{a}\right)^2 = \left\{\left(\frac{a}{a}\right)(\pm y\%)\right\}^2 = (y\%)^2 \quad (8.1)$$

$$C_{R_w} = \left(\frac{\sigma_{R_w}}{R_w}\right)^2 = \left\{\left(\frac{R_w}{R_w}\right)(\pm y\%)\right\}^2 = (y\%)^2 \quad (8.2)$$

$$C_{R_t} = \left(\frac{\sigma_{R_t}}{R_t}\right)^2 = \left\{\left(\frac{R_t}{R_t}\right)(\pm y\%)\right\}^2 = (y\%)^2 \quad (8.3)$$

$$C_\Phi = \left(m \cdot \frac{\sigma_\Phi}{\Phi}\right)^2 = m^2(y\%)^2 \quad (8.4)$$

$$C_m = [\ell n(\Phi)\,\sigma_m]^2 = [\ell n(\Phi^m)]^2 (y\%)^2 = m^2[\ell n(\Phi)]^2 (y\%)^2 \quad (8.5)$$

$$C_n = [\ell n(S_w)\,\sigma_n]^2 = [\ell n(S_w^n)]^2 (y\%)^2 = n^2[\ell n(S_w)]^2 (y\%)^2 \quad (8.6)$$

H. C. Chen and J. H. Fang.
Sensitivity Analysis of the Parameters in Archie's Water Saturation Equation. The Log Analyst. Sept – Oct 1986

Figure 3

•The ***relative impact of porosity upon the uncertainty in Sw, is a function of the cementation exponent "m"***

•Since ***"m" > 1.0,***

•$C_{Phi} > C_a$

•$C_{Phi} > C_{Rw}$

•$C_{Phi} > C_{Rt}$

Equal y% Uncertainties

•<u>Time and money are better spent on improved porosity estimates</u>, rather than than "a", "R_w" or "R_t"

•The ***relative importance of "m" and "n" require additional considerations***

$$\boxed{C_a} = \left(\frac{\sigma_a}{a}\right)^2 = \left\{\left(\frac{a}{a}\right)(\pm y\%)\right\}^2 = (y\%)^2 \quad (8.1)$$

$$\boxed{C_{R_w}} = \left(\frac{\sigma_{R_w}}{R_w}\right)^2 = \left\{\left(\frac{R_w}{R_w}\right)(\pm y\%)\right\}^2 = (y\%)^2 \quad (8.2)$$

$$\boxed{C_{R_t}} = \left(\frac{\sigma_{R_t}}{R_t}\right)^2 = \left\{\left(\frac{R_t}{R_t}\right)(\pm y\%)\right\}^2 = (y\%)^2 \quad (8.3)$$

$$\boxed{C_\Phi} = \left(m \cdot \frac{\sigma_\Phi}{\Phi}\right)^2 = m^2(y\%)^2 \quad (8.4)$$

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$$\boxed{C_n} = [\ell n(S_w)\,\sigma_n]^2 = [\ell n(S_w^n)]^2 (y\%)^2 = n^2[\ell n(S_w)]^2 (y\%)^2 \quad (8.6)$$

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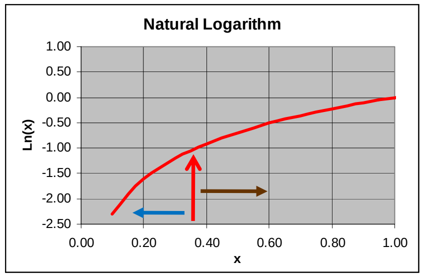
In general each attribute will have an individual uncertainty. For illustrative purposes, and to maintain contact with Chen (1986), consider the following.

- "a" = 1.0, y_a = 0%
- R_w = 0.02, y_{Rw} = 4.4%
- R_t = 40, y_{Rt} = 1%
- Phi = 0.20, y_{Phi} = 15%
- "m " = 2.0, y_m = 10%
- "n" = 2.0, y_n = 5%

A ***spreadsheet formulation*** (Figure 1) ***allows one to not only easily perform the calculations, but to also consider what the effect of a change in reservoir quality (porosity) would mean*** (because the importance of 'm' and 'n' is linked to porosity), with locally specific values.

Figure 4

•Because ***Ln(0.367) = -1***, ***both C(m) and C(n) exhibit <u>inflection points</u>***



•If **0.367 < x** < 1.00, **Abs[Ln(x)] < 1**

•If 0 < **x < 0.367**, **Abs[Ln(x)] > 1**

$$C_a = \left(\frac{\sigma_a}{a}\right)^2 = \left\{\left(\frac{a}{a}\right)(\pm y\%)\right\}^2 = (y\%)^2 \quad (8.1)$$

$$C_{R_w} = \left(\frac{\sigma_{R_w}}{R_w}\right)^2 = \left\{\left(\frac{R_w}{R_w}\right)(\pm y\%)\right\}^2 = (y\%)^2 \quad (8.2)$$

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$$C_m = [\ell\eta(\Phi)\,\sigma_m]^2 = [\ell\eta(\Phi^m)]^2\,(y\%)^2 \quad (8.5)$$
$$= m^2[\ell\eta(\Phi)]^2\,(y\%)^2$$

$$C_n = [\ell\eta(S_w)\,\sigma_n]^2 = [\ell\eta(S_w^n)]^2\,(y\%)^2 \quad (8.6)$$
$$= n^2[\ell\eta(S_w)]^2\,(y\%)^2$$

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At 20 pu formation evaluation, with the above conditions, should focus on improved porosity and 'm' estimates, with 'n' of relatively less importance. ***If porosity rises to 30 pu***, however, improved porosity estimates become more important with 'm' and 'n' having similar, and less, impact. ***As porosity drops*** below 20 pu, it is the pore connectivity ('m') that begins to dominate the accuracy.

If the water were fresher, say Rw = 0.2 instead of 0.02, 'n' diminishes in importance (Figure 5) as compared to both the amount of porosity, and its connectivity ('m').

Yet more details and variations are to be found in Chen & Fang (1986); ***coding the equations to a spreadsheet will allow application of the concept to locally specific conditions***.

Figure 5

•At **20 pu**, formation evaluation should focus on improved porosity and 'm' estimates, with 'n' of relatively less importance.

Light Green Cells require User Specification			
Light Blue Cells are calculated results			
Attribute	Individual Uncertainty	Best Estimate	Relative Un On Sw(Arch
a	0.0%	1.00	0.0000
Rw	4.4%	0.02	0.0019
Phi	15.0%	0.20	0.0900
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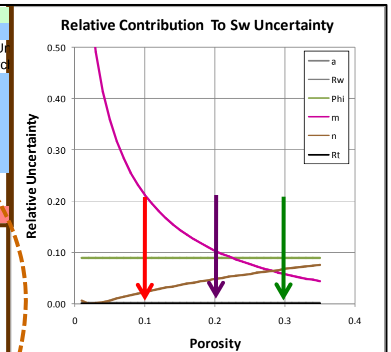
Rt	1.0%	40.00	0.0001
Sw		11%	
Sw^n		1%	
0.367 is a logarithmic inflection point			



•If porosity rises to **30 pu,** however, improved porosity estimates become more important with 'm' and 'n' having similar, and less, impact.

•As porosity drops to **10 pu**, it is the pore connectivity ('m') that begins to dominate the accuracy.

•If the water were fresher, say ***Rw = 0.2 instead of 0.02, 'n' diminishes in importance as compared to both the amount of porosity, and its connectivity ('m').***

After C. Chen and J. H. Fang. Sensitivity Analysis of the Parameters in Archie's Water Saturation Equation. The Log Analyst. Sept – Oct 1986

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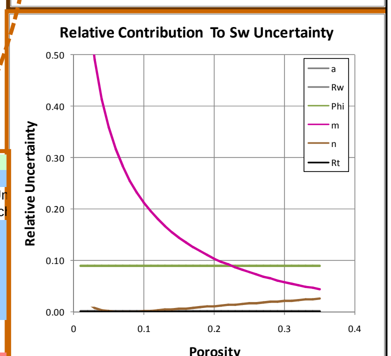
m	10.0%	2.00	0.1036
n	5.0%	2.00	0.0108
Rt	1.0%	40.00	0.0001
Sw		35%	
Sw^n		13%	
0.367 is a logarithmic inflection point			



Deterministic vs Probabilistic

Bowers & Fitz (2003) have extended Chen & Fang's work to include Sw(Dual Water), and have additionally made ***comparisons of the deterministic results to Monte Carlo Simulations***.

The ***two basic ways in which to approach the issue*** are with an equation (the analytical, or ***deterministic***, method) and with a ***statistical*** (Monte Carlo) simulation. ***Each has strengths and weaknesses***.

The analytical approach results in relatively simple equations that may be coded into a spreadsheet, or most any petrophysical s/w package. With these equations one may quickly identify the key issues for any specific situation.

- Planning (and allocating money for) a core analysis program.
- Foot-by-foot petrophysical interpretation, across a range of formation qualities and conditions.

Implicit in the deterministic method is that of a bell-shaped (Gaussian, or Normal) distribution. In actual fact, naturally occurring phenomena may in fact exhibit a different (non-Gaussian) distribution: Limpert, et al (2001).

Excel, and a variety of statistical packages, offer alternative distributions which may be used as input to the S_w calculation (Archie, or otherwise). Repeating the calculation multiple times with random selections from the various, specific input distributions (porosity, 'm', 'n', etc) will yield an output distribution, which may then be examined for characterization of the resulting uncertainty.

Normality

Many routine statistical analyses are based upon the ***assumption of a 'normal' or 'Gaussian' distribution***, and while this is a reasonable starting point, we also realize that it's not necessarily the actual distribution.

The implications of actual versus assumed distribution are addressed by Hill & Lewicki (2007) who point out that ***in many cases a normal distribution-based test can be utilized if one simply ensures that the size of the sample population is sufficiently large***. This conclusion is based on the principle which is largely responsible for the popularity of tests that are based on the normal function: as the sample size increases, the shape of the sampling distribution (distribution of a statistic from the sample) approaches normal, even if the distribution of the variable in question is not normal.

Hill & Lewicki illustrate the concept with an animation showing a series of sampling distributions (created with gradually increasing sample sizes) based upon a variable that is clearly non-normal (distribution of values is skewed). As the sample size increases, the shape of the sampling distribution becomes normal and at n ~ 30, the shape of that distribution is "almost" perfectly normal (http://www.statsoft.com/textbook/stathome.html). The principle is called the central limit theorem, and the StatSoft site is well worth taking a look at.

Monte Carlo Evaluation

Monte Carlo evaluation is based upon repeated random sampling of the various user-specified input distributions, which are then used to calculate a composite result. It's a particularly attractive approach when it is infeasible or impossible to compute an exact result with a deterministic algorithm.

Each of the individual input attributes (R_w, Phi, 'm', etc) can be represented by the appropriate Best Estimate value, encompassed by a distribution of associated Possible values, according to the appropriate user-specified distribution (Gaussian, Square, Triangular, Log Normal, etc). The individual, randomly selected results for R_w, Phi, 'm', etc are input to the Sw calculation, multiple times, and the resulting most likely value, and uncertainty distribution, determined.

While there are a number of commercial Excel-based Monte Carlo simulators, it's also straight-forward to custom code a specific application, from scratch. This option will be dealt with in our next article: ***Rolling The Dice***.

Dealing With Risk

With a rigorous mathematical relation in hand, ***we are now able to quantitatively estimate the uncertainty in the ultimate S_w estimate, once we characterize the individual input parameter uncertainties. This parameter specification is itself, a challenge***, and again subject to that original qualification of ***'one size does not fit all feet'***.

At the simplest level, repeat core analyses can be drawn upon; Hook (1983), thank you to George Eden, BP Canada, for bringing this article to my attention. Voss (1998) comments on determination of uncertainty ranges as does Bowers (2003). ***Summary points*** include

- A single interpreter should avoid making estimates on their own.
    - A single interpreter often lacks the needed knowledge to correctly estimate every parameter.
    - In addition, many interpreters have a bias that smaller errors are better and they will appear more knowledgeable about the subject.
- The error must reflect the level of knowledge about the parameters and the data quality.
- ***A standard set of uncertainty ranges must be avoided*** because there is no standard situation in which to apply them.
- ***Unusual events also pose special problems***.
    - Most people have a better recall of unusual events
        - Therefore a tendency to overestimate the probability of such an event
        - Especially if that event occurred recently
- Another very common mistake is to allow a very small amount of data to quantify the range of uncertainty
    - If data sets are small, the ranges probably need to be increased.
- Boundary Conditions

- Water saturation must lie between zero and one
- If the saturation values are too large or too small, the "best guesses" and ranges must be reconsidered and calculations remade.

- ***The final and probably most difficult problem to overcome is the culture and preconceived ideas of an organization.***
  - ***Methods and ranges of uncertainty applied to any analysis must be questioned every time they are applied.***

We should not lose sight of the fact that the porosity estimate is the result of a transform, which itself involves uncertainty (remember that old phrase, ***the devil is in the details***?). Likewise, the S_w model involves basic assumptions. For example, if clay conductivity is an issue, an Sw(Archie) evaluation has been immediately compromised.

In the carbonate world, as compared to clastic, clay is much less commonly a problem but ***dual porosity systems, particularly in the transition zone, can challenge the Archie equation electrical circuit model***: Griffiths et al (2006), Figure 6. High in the oil column, or gas wells in general, increased capillary pressure will cause the hydrocarbon to more efficiently displace the brine, and the magnitude of the problem diminishes.

Carlos Torres-Verdin cautions "my experience shows that the ***biasing of apparent resistivity curves*** due to post-processing techniques (e.g. deconvolution) could be more detrimental to uncertainty than Archie's parameters. The most conspicuous cases is the one of a thin, hydrocarbon-saturated bed, where bed thickness and invasion can give you much more uncertainty grief than Archie's parameters".

•***Carbonate pore structure is complex*** with a ***wide variety of pore sizes*** ranging from visible to microscopic.

•***As hydrocarbon charging occurs***, ***macro-pore water*** tends to be ***displaced first.***

•***Depending on*** the ***buoyancy pressure and viscosity of the oil***, ***a portion of the meso-pores are likely to become oil charged.***

•While the ***Archie equation*** has had tremendous success, there are ***limitations in carbonates***

•One of the critical ***assumptions*** typically made is that ***the measure current moves uniformly through the formation***

•The presence of ***water-filled micro-pores in close proximity to the larger hydrocarbon-filled pores may short-circuit the resistivity measure currents***

•This causes the ***oil saturation*** to be ***under-estimated***

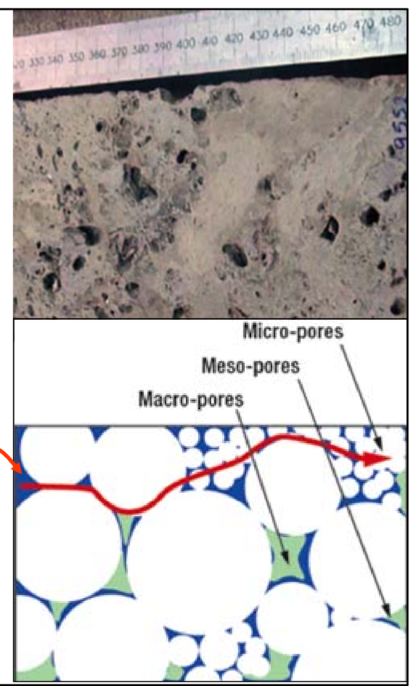


Figure 6

Estimating S_w with a volume measurement
R. Griffiths, A. Carnegie, A. Gyllensten, M. T. Ribeiro, A. Prasodjo, and Y. Sallam. World Oil, October 2006

Finally, there is always the possibility ***Good Luck can save the day***. As a young man just home from the Army, and attending Missouri State University, three men stimulated my interest in Physics and Applied Mathematics. In the intervening 40 years, I've ***often thought of how very lucky I was to have my path, cross theirs.***

- Banks, Dr Larry
- Schmidt, Dr Bruno
- Sun, Dr Woodrow

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Biography

R. E. (Gene) Ballay's 32 years in petrophysics include research and operations assignments in Houston (Shell Research), Texas; Anchorage (ARCO), Alaska; Dallas (Arco Research), Texas; Jakarta (Huffco), Indonesia; Bakersfield (ARCO), California; and Dhahran, Saudi Arabia. His carbonate experience ranges from individual Niagaran reefs in Michigan to the Lisburne in Alaska to Ghawar, Saudi Arabia (the largest oilfield in the world).

He holds a PhD in Theoretical Physics with double minors in Electrical Engineering & Mathematics, has taught physics in two universities, mentored Nationals in Indonesia and Saudi Arabia, published numerous technical articles and been designated co-inventor on both American and European patents.

At retirement from the Saudi Arabian Oil Company he was the senior technical petrophysicist in the Reservoir Description Division and had represented petrophysics in three multi-discipline teams bringing on-line three (one clastic, two carbonate) multi-billion barrel increments. Subsequent to retirement from Saudi Aramco he established Robert E Ballay LLC, which provides physics - petrophysics consulting services.

He served in the U.S. Army as a Microwave Repairman and in the U.S. Navy as an Electronics Technician, and he is a USPA Parachutist and a PADI Dive Master.

