GeoNeurale

MACHINE LEARNING ALGORITHMS APPLICATIONS IN SEISMIC AND PETROPHYSICAL ANALYSIS

Part 2 : NON-LINEAR HYPOTHESES IN SEISMIC AND PETROPHYSICAL SYSTEMS

Unless we have formations with homogeneous and isotropic textures and mineral systems and same spatial properties for geological architectures we will encounter non-linear relations between input and target attributes. This is the normal situation in all seismic and petrophysical problems. This issue could trigger a discussion on geostatistical methods in which local properties can always be abstracted from the macrosystem and assume a linear simulation behavior.

Non-linear systems in geophysics are used for classification and mapping.
We can have binary and multiclass classification (where multiclass classification can also be fed into an output database for discrete parametric quantification).
Logistic operators build up basic units that can be extended into neural systems which can learn most complex non-linear hypotheses.

In critical problems encountered in geophysics it is important to fill database of interpretation parameters where measurements are not available. For instance the calculation of a target log from other input logs, the calculation and spatial distribution of petrophysical properties in the seismic attributes volume.

Classification is extremely important for reservoir characterization.
Binary classification can predict a formation based on an input of petrophysical, seismic attributes with eventual addition of geological and structural constraints.
Multiclass classification can discriminate between target formations or attributes based on inputs of measured parameters and calculated petrophysical and seismic attributes.

Classification is a probabilistic quantification that an event will be verified.
The convention is that an output will be positive when equal to 1 , or negative when equal to 0.

For theses issues the transition from linear into non-linear regression is a main step for further development of algorithms to compute complex features and learn the behavior of a geological system.

Originally classification problems were solved by linear regression, for instance distributing a sample dataset of a single feature and computing the linear hypothesis which minimizes the cost function, obtaining the slope of the regression line. Then choosing the point y= 0.5 as discriminating point between 0 and 1.
As a very simplified example we could think **x = Shy/Sw** and **y = Prob (Pay)** for oil exploration or for deep geothermal reservoirs **x = Flow Rate** and **y = Prob (5 MWatt)** (Fig. 1).

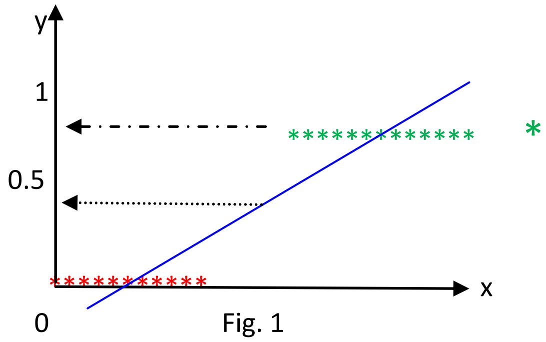

Fig. 1

However using this solution the results could be altered by the presence of outliers (*) which can not be discarded and have to be included.

The solution to this problem consists in minimizing the weight of outliers.
Therefore a new hypothesis function has to be used that will cover values of x between $-\infty$ and $+\infty$: $-\infty < x < +\infty$ and y between 0 and 1 : $0 < y < 1$, defining y as probability that the event x will be verified.

This problem can be solved by using a sigmoid or logistic function of equation :

Eq. 1

$$g(z) = \frac{1}{1+e^{-z}}$$

where:

$$z = (\theta^T x)$$

which can be also written as :

$$g\,(z) = \frac{1}{1+e^{-\theta^T \mathrm{x}}}$$

then the hypothesis of the logistic regression for classification is:

$$\mathrm{h}_{\theta\ LOG}\ (x) \ = \ \mathrm{g}\,(z)$$

Where the subscript LOG means that h_q is not a linear hypothesis but a logistic hypothesis.

The graphic of the sigmoid or logistic function is shown in Fig. 2 .

The logistic equation maps the linear regression including and minimizing outliers effect into the probabilistic sigmoid function.

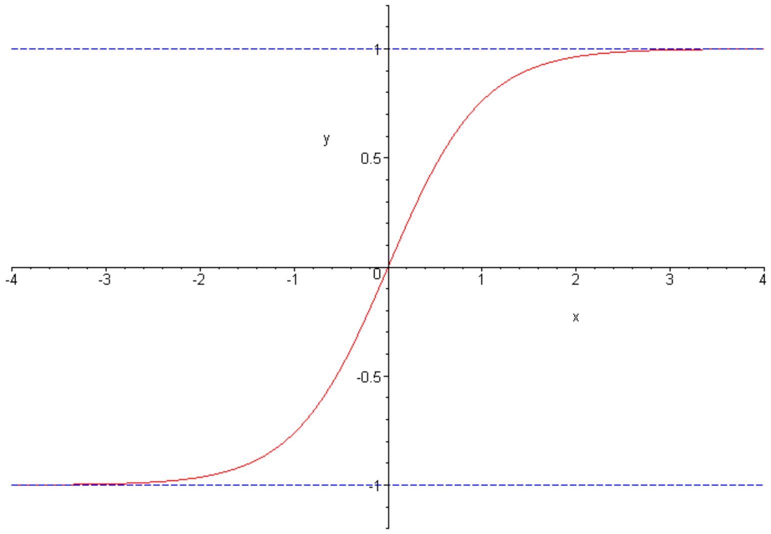

Fig 2

The use of a logistic function transforms a linear regression into a probabilistic output in which the sigmoid function is the new hypothesis that produces a probabilistic output P(y = 1$|x;\theta$) :
P = probability that y = 1 given x parametrized by θ .

P(y = 1$|x;\theta$)   where →   P(y = 1 | $x;\theta$) + P(y = 0 | $x;\ \theta$) = 1

OBJECTIVE FUNCTION

If we used the same cost (objective) function as in the linear regression, then due to the exponential term at the denominator we would get a non-convex function behavior, and the search of a minimum point would not be possible (Fig 3) .

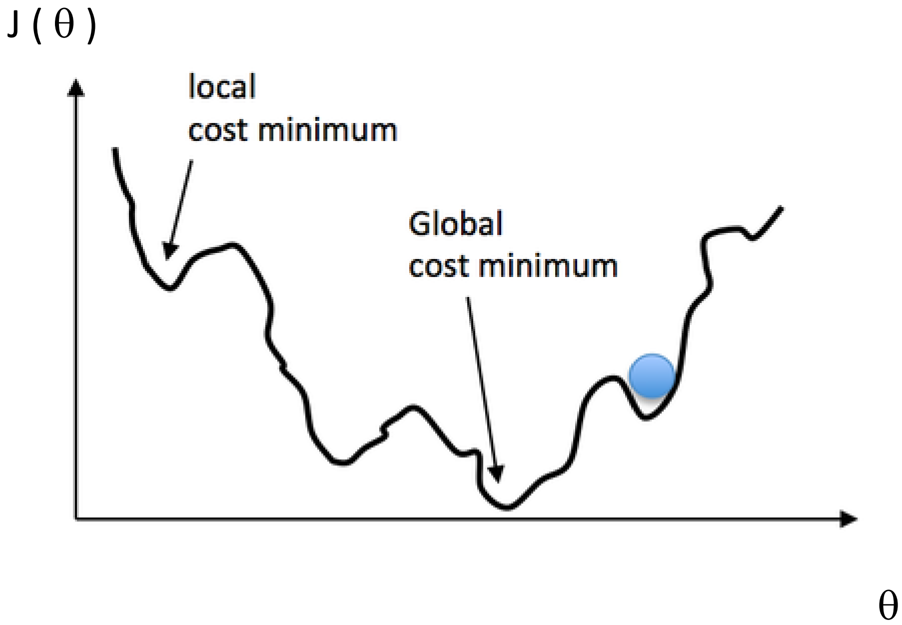

Fig. 3 2D Non-Convex function

Therefore a new cost function derived from the sigmoid presenting a convex behavior have to be used (Eq. 4) .

Eq. 4

$$J(\theta) = -\frac{1}{m}\left[\sum_{i=1}^{m} y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log\left(1 - h_\theta(x^{(i)})\right)\right]$$

This is the standard cost function of the logistic regression,
where m is the number of training samples, (i) is the index of the ith training sample for one feature, x is the input attribute and y the target.

$J(\theta)$ is equivalent to the equation 5 :

Eq. 5

$$\left.\begin{array}{ll} -\log\ \mathrm{h}_{\theta}(x^{(\mathrm{i})}) & \text{if y=1} \\ -\log\ (1-\mathrm{h}_{\theta}(x^{(\mathrm{i})})) & \text{if y=0} \end{array}\right\} \rightarrow J(\theta)$$

In fact substituting y=1 or y=0 to Eq. 4 we get the respective solution of Eq. 5

This function has the property of being convex and its graphic assumes two different aspects depending on the value of y.

For y =1 the function presents a concavity in the right direction (Fig. 4).

For y =0 the function presents a concavity in the left direction (Fig. 5).

$\mathrm{h}_{\theta}\ (x^{(\mathrm{i})})$ is the hypothesis of the logistic function.

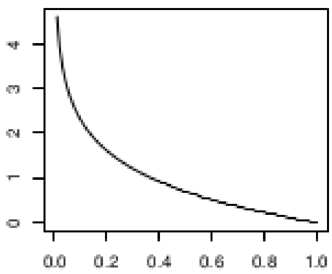

Fig 4

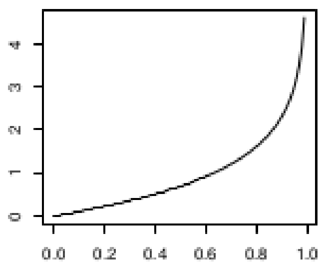

Fig. 5

The extension of logistic regression are logical systems which can learn from a training set and compute complex non-linear hypotheses that can predict the behavior of geological systems.

Logistic regression units are standard building blocks of neural networks layers.
Each neural unit is a logistic regression operator which is a decision-maker for the next input layer. The decision will be constrained in the input layer and in the output layer by the training set during the learning phase, but the learning logic and solution of a learning process is enclosed into the parameters (or weights) θ .

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