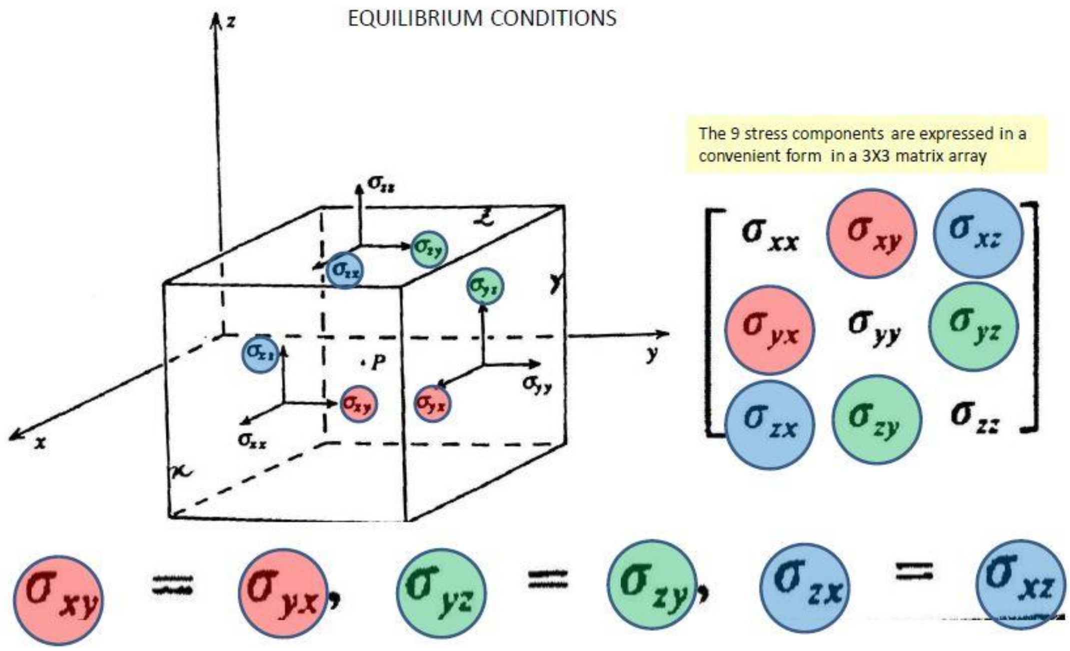
VERTICAL TRANSVERSE ISOTROPY AND AVO Rs REFLECTIVITY

The original formulation of the stress distribution on an elementary cube as a 3x3x3x3 tensor has allowed originally to calculate the directional dependence of P and S waves velocity in anisotropic media.
In a equilibrium situation the 9 stress components are reduced to 6.



This permitted to simplify calculations of relations between stress and strain.
Applying Hooke's law to the material under stress, mean to apply a 3x3 tensor multiplication to each elemental stress component for 3 faces of the elementary cube (resulting in a 3x3x3 tensor) and re-dimension (turn as vector rotation and change scalar value) to transform stress into the corresponding strain. For 3 faces of a cube a 3x3x3x3 tensor.

After the introduction of Voigt's notation and reducing the 3x3x3x3 tensor to a 6x6 matrix of independent variables for the spatial solution of Hooke's law, it became possible to simplify, better visualise and study the direct interdependence between stiffness coefficients and seismic waves velocities.
The linear proportionality between stress and strain contained within the Hooke's law is expressed by the stiffness parameters.

$$\sigma_1 = c_{11}\varepsilon_1 + c_{12}\varepsilon_2 + c_{13}\varepsilon_3 + c_{14}\varepsilon_4 + c_{15}\varepsilon_5 + c_{16}\varepsilon_6$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{12} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{13} & c_{23} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{14} & c_{24} & c_{34} & c_{44} & c_{45} & c_{46} \\ c_{15} & c_{25} & c_{35} & c_{45} & c_{55} & c_{56} \\ c_{16} & c_{26} & c_{36} & c_{46} & c_{56} & c_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{bmatrix}$$

In the wave equation the stress formulation for the Hooke's law as a function of the strain and stiffness tensor is substituted to the stress parameter.
This better visualizes the dependency of velocities, amplitude and their polar and azimuthal anisotropy from stiffness coefficients.
This dependency is formulated by Thomsen coefficients:

$$\varepsilon = \frac{C_{11} - C_{33}}{2\,C_{33}}$$

$$\delta = \frac{(C_{11} + C_{44})^2 - (C_{33} - C_{44})^2}{2\,C_{33}\,(C_{33} - C_{44})}$$

$$\gamma = \frac{C_{66} - C_{44}}{2\,C_{44}}$$

Solving the wave equation with the VTI constraints of the stiffness matrix, means retransforming the 6x6 matrix into the 3x3x3x3 tensor.
The solution is a group of 3 waves at different velocities and polarizations: P, S Longitudinal and S Transverse.

In the context of the Zoeppritz equation linearization, Aki-Richards introduced the AVO reflectivity equation in terms of Elastic Parameters and Weights.

AVO P-Reflectivity

Aki-Richards (Wiggin formulation) Rep. Hampson-Russell

$$R_{PP}(\theta) = A + B\,Sin^2\theta + C\tan^2\theta\,\sin^2\theta$$

A = Intercept
B = Gradient
C = Curvature

Shuey, Wiggings et. al. reformulated Aki-Richards equation for P waves in terms of zero offset reflectivity, gradient and curvature.

Starting from Aki-Richards formulation on S waves, Thomsen introduced the AVO equation with polar and azimuthal dependence of Reflectivity. For the simple case only the VTI solution have been considered not the orthorombic case.

AVO S-Reflectivity

Thomsen

$$R_{PS}(\theta) = \sin\theta\,(B_{PS} + C_{PS}\sin^2\theta)$$

Gradient

$$B = ½\,[\,\Delta\rho/\rho - (2/\gamma_0)\ \Delta\mu/\mu + (\gamma_0/(1 + \gamma_0)\,\Delta\delta\,]$$

Curvature

$$C = -½\,\gamma_0^{\,2}\,[\,\Delta\rho/2\rho - (2+\gamma_0)\,\Delta\mu/\mu\,] + (\gamma_0 - 1)\,\gamma_0\Delta\eta$$

$$+ ½\,[\,2\gamma_0^{\,3} + (\gamma_0 - 1)^2\,] / [\,2\gamma_0\,(\gamma_0^{\,2} - 1)\,]\ \Delta\delta$$

Effects of polar and azimuthal dependency can be visualized and studies with use of the Garotta Cross, introduced in the context of seismic Multicomponent studies (anisotropy circle).

GeoNeurale Researches the interrelation of micro and macro-field properties in the formulation of stiffness coefficients and Thomsen anisotropy coefficients.

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